Zero Thrust Velocity Vector Control for Interstellar Probes: Lorentz Force Navigation and Circling

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The fundamental equations and conditions governing the interaction of a charged interstellar probe with the magnetic fields existing in interstellar space are examined. Since the ion densities in interstellar space are very low, a substantial voltage can be easily maintained. Because of the large distances and long times involved, it is found that the weak forces from the interaction of the interstellar magnetic field with the charge on the probe can build up an appreciable change in the direction of the velocity vector of the probe. This change is found to be large enough to allow for extensive midcourse corrections on a multi-ton, one-way charged probe or even to return a smaller probe to its starting point. Thus, the propulsive energy required to launch a nonreturn fly-by probe to the nearest stars is also sufficient for a round trip probe.

Introduction

THERE is a long and venerable history of published papers[†] that have studied the various problems connected with interstellar flight, nearly all of them on the fundamental problem of the design and operation of the propulsion mechanism.

Since the design or construction of even the smallest interstellar probe is a number of decades in the future, any discussion of the use or operation of such a probe should be carried out at the level of defining the physical and practical limitations. It is in this spirit that we examine the use of charge to control the direction of the velocity vector of a probe by interaction with the interstellar magnetic field. Because many of the parameters (for instance, the strength of the interstellar magnetic field) are only known to an order of magnitude, the calculations will be carried to only 20% accuracy.

We will not get into the realm of relativistic velocities since the relativistic corrections become larger than 20% only when $v/c > 0.5$. At this velocity so many other effects become important that it is not feasible to consider them in this study.

In order to change the course of an interstellar probe after it has built up the necessary speed, it is only necessary to change the direction of the velocity vector. Since the magnitude does not have to be changed, there is no real reason for the expenditure of energy to make the correction. But we must also conserve momentum, and if we use rockets for the correction maneuver, their method of operation requires the expenditure of considerable amounts of mass and energy, which must be included in the vehicle weight.

If, however, we can "push" against the universe itself, with its infinite mass, we can make the necessary changes with a negligible amount of energy. One of the objects in the universe which we can utilize is the interstellar magnetic field. There are a number of ways to interact with a magnetic field. We could use the Lorentz force on a moving charge, the eddy currents induced in a moving conductor, the Meissner effect repulsion of a superconductor, or the magnetic gradient force on a permanent magnetic dipole. The one considered in this paper is the Lorentz force.

Received January 18, 1963; revision received February 3, 1964

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† The author has a bibliography of approximately 200 references pertaining to interstellar travel. A copy is available upon request.

Lorentz Force

When a charged object moves through a magnetic field, it experiences a force at right angles to both its direction of motion and the magnetic field. This Lorentz force is given by the equation

$$\mathbf{F} = Q(\mathbf{v} \times \mathbf{B})$$

where $Q \equiv$ charge on the body, $\mathbf{v} \equiv$ velocity of the body, and $\mathbf{B} \equiv$ magnetic field strength. If the magnetic field is uniform, then the charged object will travel in a helical path. If the initial motion of the particle is at right angles to the magnetic field, then the helix will degenerate into a circular orbit. The radius of the orbit is determined by the mass and velocity of the object since the Lorentz force will be balanced by the inertial force:

$$(Mv^2/R)(\mathbf{R}/R) = Q(\mathbf{v} \times \mathbf{B}) \tag{1}$$

Since the important parameter is the minimum radius of curvature, which determines the size of the minimum return circle or the maximum maneuvering ability, we shall write the preceding equation in the form

$$R = Mv/QB \tag{2}$$

where we have assumed that the velocity of the charged object is at right angles to the magnetic field. Thus, for any given magnetic field and velocity, the radius of curvature is determined by the sign and size of the charge-to-mass ratio.

In order to use this force in space effectively, it is necessary to find an efficient lightweight method of maintaining a substantial charge on a space vehicle in spite of the discharging effects due to field emission and ion capture from the surrounding regions. It is shown in the following sections that the concept is quite feasible for probes or vehicles in interstellar space, whereas it would not work in interplanetary space because of the high ion densities near the sun. By using a long, thin quartz fiber to increase the capacitance of the probe, the charge-to-mass ratio can be made very large without having to use high voltages. This, in turn, means that the necessary voltage and current can be obtained from a few grams of a suitable radioisotope or a very small charged particle accelerator.

The Interstellar Environment

The utilization of this concept depends upon the existence of an interstellar magnetic field. As yet there is no firm knowledge of the magnetic fields away from the stars, but in a recent survey of all the evidence, Woltjer[1] came to the conclusion that the best estimate for the average magnetic field of the galaxy is $3 \times 10^{-5}g$ (3×10^{-9} weber/m^2) We shall use this value in this paper Of course, to use this force for navigation, we would have to know the strength and direction of the interstellar magnetic field over the path to be traveled It is for this reason that it would be useful to send simple charged probes out of the solar system to map the interstellar fields

The other property of interstellar space which will be of interest is the density and composition of interstellar matter From the various estimates made by astronomers[2] we will assume that in the interstellar space around the sun, the interstellar matter has the following properties: density of neutral hydrogen, 1 atom/cm^2 = 10^6 atoms/m^3; density of ions, 10^{-3} ions/cm^2 = 10^3 ions/m^3; temperature (kinetic), 100°K; and maximum rms velocity (turbulence), 10 km/sec = 10^4 m/sec We will neglect other low density components such as dust grains or atoms other than hydrogen

Self-Capacitance

With the magnetic field determined by nature, and the probe velocity determined by propulsion limitations and mission requirements, the only parameters that we can vary are the charge and mass of the probe Since we will be concerned here with macroscopic bodies, which will have to be charged artificially, the amount of charge that can be held on the body will be determined by the capacitance of the probe and the voltage induced:

$$Q = CV \tag{3}$$

A body isolated from all other objects still has a capacitance associated with it This self-capacitance depends upon the linear dimensions of the body and a shape factor The general formula is

$$C = 2\pi\epsilon k L \tag{4}$$

where $L \equiv$ length or diameter of the object, $\epsilon \equiv$ capacitivity of the medium (for free space $\epsilon = 8\ 85 \times 10^{-12}$ coul^2-sec^2/kg-m^3), and $k \equiv$ factor that depends upon the shape For a sphere $k = 1$, for a thin disk $k = 2/\pi$, and for a single long cable $k = 1/\ln(2L/d)$ For a long, thin wire, as L/d varies from 10^5 to 10^{11}, or 6 orders of magnitude, the reciprocal log varies a factor of two from 0 08 to 0 04

Cook has looked into the problem of the charging of space vehicles and has found that a single long cable has the highest charge-to-mass ratio [3] For the purposes of this study, we will choose such a configuration, and to simplify calculations, we will assume that $k = 0\ 06$ Thus, the charge on a vehicle whose capacitance is determined by a long cable is, in mks units,

$$Q = CV = 2\pi\epsilon k L V \approx 3 \times 10^{-12} LV \tag{5}$$

Total Probe Mass

We have seen that the total charge of the probe depends, in an obvious sort of way, on the length of the charged cable and the voltage induced In a less obvious way, we also find that the total mass of the probe also depends upon these parameters Besides the payload mass M_p, we must include the mass of the voltage source M_v and the mass of the cable M that determines the self-capacitance of the probe:

$$M = M_p + M_v + M_c \tag{6}$$

The mass of the voltage source will depend not only upon the voltage needed, but also on the current drain due to the attraction of charged ions from the surrounding space This current, in turn, depends upon the voltage, the velocity, and the ion density The mass of the cable is determined by the length and diameter required, and the diameter of the cable is limited by the electrostatic forces tending to pull the charged cable apart

By investigating all these parameters and their interactions with each other, and by imposing engineering limitations, one can optimize them, and, by means of parametric studies, determine the operating regions where the basic concept is most easily implemented

Cable Mass

The mass of cable of density ρ, length L, and diameter d is just

$$M = \rho L A = (\pi\rho/4) L d^2 \tag{7}$$

However, we cannot make the cable too thin or the electrostatic forces will tear it apart If the cable has a tensile strength of T, then the minimum area of the cable is determined by the electrical force from the charge Q:

$$TA \geq F = (Q/2)^2/4\pi\epsilon L^2 \tag{8}$$

where we have assumed that the charge is divided between the two ends of the cable Because the charge is a function of the capacitance, which, in turn, depends upon the length of the wire [see Eq (5)], the minimum area or diameter is found to be a function of the voltage only:

$$A = (\pi/4)d^2 \geq (\pi\epsilon k^2/4T)V^2 \tag{9}$$

$$d = (\epsilon/T)^{1/2}\, kV \tag{10}$$

Thus, the mass of the cable has a minimum determined by the voltage applied and the ratio of the density to tensile strength It is for this reason that a metal coated glass cable will be better than a steel wire:

$$M_c = \rho L A \geq (\pi\epsilon k^2 \rho/4T) V^2 L \tag{11}$$

However, we also have a practical lower limit in that it is difficult to draw a quartz fiber less than $10\mu(10^{-5}$ m) in diameter A more practical equation for estimating the cable mass would be

$$M_c = 1\ 5 \times 10^{-7}(1 + 10^{-12}V^2)L \tag{12}$$

in mks units, where we have assumed that $\rho = 2000$ kg/m^3 (2 g/cm^3), and $T = 3 \times 10^8$ newtons/m^2 (40,000 psi) From Eq (12) it is seen that the electrostatic forces will become important for an applied voltage V greater than 10^6 v

Interaction with Interstellar Ions

Since space is a conductor full of mobile ions and electrons, there will be a strong interaction between the charged cable and the interstellar environment This will give rise to a number of possible effects that could prevent the use of the Lorentz force for velocity vector control Some obvious ones are magnetic shielding effects due to the formation of a plasma sheath, drag effects due to coulomb interactions, and discharging effects due to ion capture Because of the very unusual properties of the assumed environment and the proposed systems any analysis of the interaction of the charged cable with the interstellar ions will involve some very unusual aspects of plasma physics For instance, one might suppose that a Debye sheath could form that will shield the charged cable from the interstellar magnetic field, and if we assume that the usual analysis is valid, a typical Debye length is approximately 20 m However, the usual analysis assumes that the electrons are in thermodynamic equilibrium with respect to the charge that they are shielding This is not necessarily true here since we have assumed that the charged interstellar probe is moving from 10^5 to 10^8 m/sec, and the thermal velocities of the interstellar ions and electrons are much lower than this Also, the potential energy of the ions and electıons anywhere near the charged probe, because of the coulomb interaction, is much greater than the thermal kinetic energy, but this is further complicated by the pıesence of the inteıstellar magnetic field that gives an appreciable anisotropic rigidity to the motion of the interstellar ions

A detailed analysis of the plasma-type interactions is beyond the scope of this paper and will be presented in a future study; however, it is felt that they will not present a major problem This feeling is based on the fact that, in most cases, the charged cable is not a minor perturbation in an interstellar plasma calculation, but the dominating component of the problem For instance, the total charge on a 100 km cable at 5×10^5 v is 0 15 coul [Eq (5)] And, since the interstellar ion density is only 10^3 ions/m^3 or $1\ 6 \times 10^{-16}$ coul/m^3, this means that there is as much charge on the cable as there is in all the space within 100 km

It is conceptually possible that a charge sheath might build up around the charged cable and be carried along with it, for it is known[4 5] that stable orthogonal orbits exist in a logarithmic potential However, this is doubtful since, from energetic considerations, the only source of these shielding charges would be the cable itself, either from partially scattered and slowed charged particles from the voltage source or from secondary emission of electrons (if the cable is charged positively) However, the paths of all these particles, since they originated on the wire, would terminate on the wire after one orbit, since the pıobability of scattering in interstellar space is so low

If, as we suppose, there would be no shielding effects, then there will be a strong coulomb interaction with the surrounding ions This will give rise to coulomb drag energy loss and discharge currents

Coulomb Drag

The problem of the drag effects on a charged wire moving through an ionized space has received a number of highly sophisticated analyses[6–8] because of the West Ford project However, the papers were based on the assumptions that the Debye length was comparable to the length of the wire, the charge on the wire was built up spontaneously, and that magnetic rigidity effects were not applicable, so that the results obtained in these studies do not necessarily apply to a charged interstellar probe

We can, however, estimate the maximum amount of energy lost if we assume that every ion that comes within the sphere of influence of the charged cable receives an amount of kinetic energy equivalent to the maximum electrostatic potential eneıgy at the cable Such estimates indicate that coulomb drag may be a problem for long, light, highly charged cables, but it will not appreciably effect the larger interstellar probes For example, a one-ton probe with a 1000 km cable at 5×10^5 v, moving with a velocity of 10^8 m/sec, would have a maximum velocity loss because of coulomb drag of only 1 5%/light year

Capture Cross Section of a Charged Cable

The calculation of the capture cross section for ions of a charged cable for a completely general case would be very complicated because of the difficult geometry If, however, we assume that the cable is moving at right angles to its axis (with its axis parallel to the magnetic field, for example), then we can estimate the cross section by using the equation for the potential of a line charge The problem then reduces to a one-dimensional central force scattering problem very similar to the more familiar $1/r^2$ problem[9] The problem is worked out in the Appendix, and for the condition that the velocity of the probe is greater than the velocity of the ions (10 km/sec), we obtain the result that the cross section is

$$\sigma = Ld[1 + (2qV/mv)]^{1/2} \tag{13}$$

where $q/m \equiv$ charge-to-mass ratio of the ion and $v \equiv$ velocity of the vehicle

The current due to charged ions intersected by the captuıe cross section of the cable is

$$I = q\sigma nv = qnvLd[1 + (qV/mv^2)]^{1/2} \tag{14}$$

where $n \equiv$ density of ions (10^3 ions/m^3)

In order to obtain a parameter that is independent of the length and diameter of the cable we will calculate the current density If the cable is charged positively, it will attract negative ions, so we will use the charge and mass of the electron in Eq (14) to obtain, in mks units,

$$J_+ = I_+/Ld = 1\ 5 \times 10^{-16}\, v(1 + 4 \times 10^{11}\, V/v^2)^{1/2} \tag{15a}$$

If the cable is charged negatively, it will attract the positively charged protons, which, because of their larger mass, have a much lower capture cross section The current density in this case is

$$J_- = I_-/Ld = 1\ 5 \times 10^{-16}\, v(1 + 2 \times 10^3\, V/v^2)^{1/2} \tag{15b}$$

in mks units These two cuırent densities are plotted in Fig 1

Notice that even for the extreme case of a long ($L = 10^6$m $= 1000$ km) and thick ($d = 100\mu = 4$ mils) cable with a positive voltage of 10^6 v, the current due to electron capture is only 9-μ amp, or a power of 9 w

Voltage Source Mass

The mass of the voltage generating apparatus is difficult to estimate since it obviously depends upon the particular type of generator In any case, it will certainly depend upon the voltage to be generated and the leakage rate expected Upon investigation, we have found that the most suitable source for generating the voltage is a radioisotope There are a fair number of suitable elements with half-lives comparable to mission times and which emit electrons, positrons, or alpha particles with energies up to a few million electron volts For example, a positron from Na^{22} has an energy of 0 5 Mev, and it is therefore capable of escaping from a potential of 5×10^5 v Thus an isolated body with a thin layer of Na^{22} would soon find itself at a negative potential of a little less than half a megavolt

The amount of radioisotope the probe will have to caıry will depend upon the discharging current due to the capture of ions from the surrounding interstellar medium that was calculated in the previous section If we assume a radioisotope with a long decay time τ and a mass per atom of w, and if we also assume that only a small percentage η of the emitted particles come off in a direction sufficiently perpen-

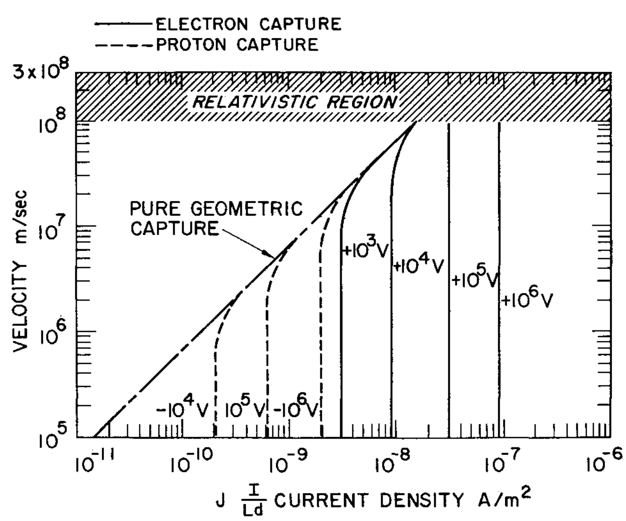

Fig 1 Discharging current vs velocity

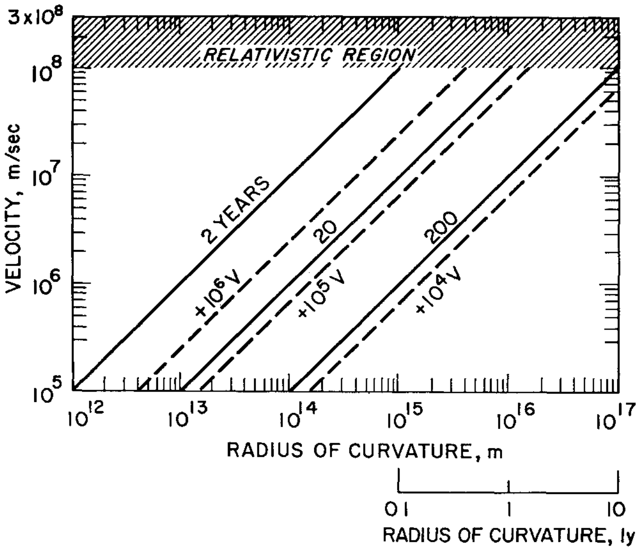

Fig 2 Radius of curvature for zero payload

dicular to the equipotential lines to enable them to escape, then the mass of a radioisotope voltage source can be approximated by

$$M_v = wI\tau/\eta q = wJLd\tau/\eta q \tag{16}$$

Let us choose some numbers for a particular radioisotope that we would all like to get rid of anyway, Strontium 90: emission, 0 531 Mev electrons, no gammas; half-life, 20 yr, decay time $\tau = 9 \times 10^8$ sec; weight, 1 5 $\times 10^{-25}$ kg/atom If we assume an efficiency of 0 01, then we get for the mass of the radioisotope in mks units

$$M_v = 10^5\, JLd \tag{17}$$

Since the minimum diameter is limited by fabrication problems and the voltage applied [see Eq (10)],

$$d = 10^{-5}(1 + 10^{-6}\, V) \tag{18}$$

the mass of a radioisotope voltage source is seen to be a function of length, velocity, and voltage

$$M_v = (1 + 10^{-6}\, V)JL \tag{19}$$

where J is a function of the velocity and voltage and must be obtained from Fig 1

Notice that even for the extreme case of a cable 10^6 m long with a voltage of 10^6 v, only a few ounces (90 g) of radioisotope is required to keep the cable charged

The voltage level of the probe can be controlled at any potential less than the maximum by a number of means such as field emission, increasing the capture cross section, or intercepting the beta particles before they leave

Parametric Study

If we now substitute into Eq (2) the estimates for the mass and charge of an interstellar probe using a thin quartz fiber cable charged by a radioisotope [Eqs (5, 12, and 19)], we obtain the approximate relation

$$R = \frac{Mv}{QB} = \frac{(M_p + M\ + M_c)v}{2\pi\epsilon kLVB}$$

$$\approx \frac{M_p + (1 + 10^{-6}V)JL + 1\ 5 \times 10^{-7}(1 + 10^{-12}V^2)L}{10^{-20}LV}\, v \tag{20}$$

in mks units where we have used the assumptions that $B = 3 \times 10^{-9}$ weber/m^2 and $k = 0\ 06$ The quantity J, which is a function of the voltage and the velocity, must be obtained from Fig 1 The interactions between these various parameters are shown by Figs 2–4

One of the parameters that must not be overlooked is the mission time For a probe on a circular flight path the mission time is given by $2\pi R/v$ If this is too long, then that particular application is not suitable since one would like to see results in a man's active lifetime Although times longer than this could be considered, it is very likely that data from an experiment more than 100 years old would have been superseded by the technological advancements in instrumentation during the flight

If we first examine the operation in the limit where the payload mass is negligible, we see that the length of the cable drops out of the equation and the radius of curvature is determined solely by the velocity of the probe and the voltage:

$$R = \frac{(1 + 10^{-6}V)J + 1\ 5 \times 10^{-7}(1 + 10^{-12}V^2)}{10^{-20}V}\, v \tag{21}$$

When this equation is plotted in Fig 2 along with the round trip time given by

$$t = 2\pi R/v \tag{22}$$

we see that as the voltage increases, the radius of curvature

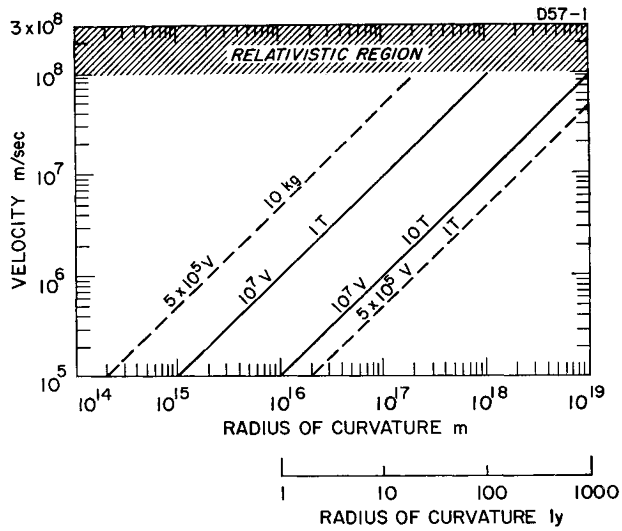

Fig 3 Radius of curvature for laige payload

decreases with a saturation setting in for voltages greater than 10^6 v This saturation is due to the increase in diameter of the cable necessary to overcome the electrostatic disruptive forces Since too low a voltage means that the round trip time becomes too long, it seems that there is an optimum voltage of 10^5 to 10^6, which is easily supplied by radioisotopes Because the equation is independent of length in this approximation, the curves are just as valid for a 1-cm fiber as they are for a fiber 1000 km long Of course, the long cable can carry a larger payload without its mass becoming a factor in the calculations

In the other limit, where the mass of the payload is assumed to be much larger than the mass of the cable or the voltage source, the equation reverts to the original one:

$$R = \frac{M_p v}{QB} = \frac{M_p v}{2\pi\epsilon kLVB} = \frac{M_p v}{10^{-20}LV} \tag{23}$$

In this approximation we are more interested in the amount of midcourse correction that we can give an unmanned probe, and a radius of curvature of 1000 light years is still interesting, since it means a substantial amount of correction capability with a negligible amount of power It should, of course, be emphasized that the correction is limited to the directions that are at right angles to the lines of magnetic force in that particular region of space The curves in Fig 3 are calculated for a cable length of $L = 10^6 = 1000$ km Although this sounds long, the cable in the low voltage case will weigh less than a kilogram and can be stored on a standard size wire spool

We can see the interplay between the various parameters in the intermediate region when we plot the radius of curvature for a number of different combinations of payload mass, velocity, and cable length for a constant potential of 5×10^5 v (Fig 4) As the mass of the payload decreases, we see that the cable length becomes unimportant and that the radius of curvature depends only upon the velocity The round-trip times indicated for negligible payload are all the same value, six years The longer the cable, the larger the payload mass can be before its mass starts increasing the radius of curvature In the limit of large payload mass or short cable length, the radius of curvature becomes a linear function of the mass

Conclusions

By examining the curves we can see that there are three regions of interest In one case, we could send out an interstellar probe with a very large payload capable of long-range

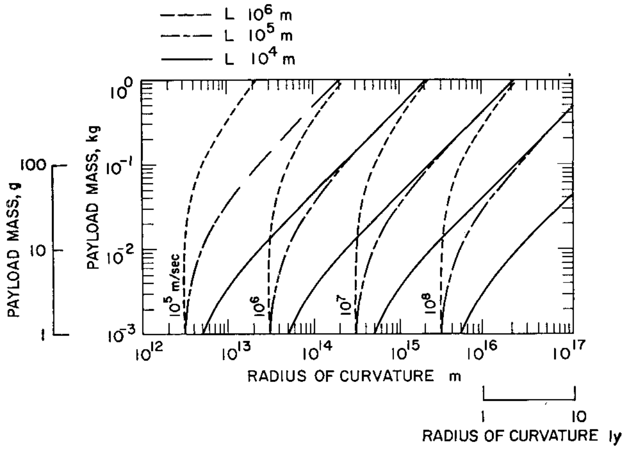

Fig 4 Payload mass as a function of v, L, R

communication and use a charged cable to provide for moderate changes in direction The charged cable could not only be used for midcourse corrections, but it could also be reused after the fly-by of the first stellar system to change the course of the probe toward another system farther out Thus, we could examine more than one star system with the same probe

In another case, we could send out an interstellar probe with a very light instrument package in a round trip that takes it past Alpha Centauri and then back into the solar system where a small transmitter would release the stored data In a third case of possible interest, we could shoot small, thin charged fibers into the outer reaches of the solar system These fibers would carry very small payloads consisting of built-in thin films or semiconductor devices with negative resistance characteristics By properly designing the position of the charging radioisotopes and the discharging field emission points, currents can be made to flow in the metal coated portions of the fiber and used to power the devices, making them oscillate The rf energy can then be radiated into space by the fiber acting as a multielement array of electromagnetic dipoles with a very large effective area By tracking these wires as they move in helical paths through space, the strength and direction of the magnetic field, and possibly even the ion density, can be calculated

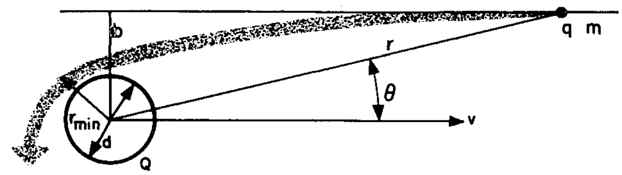

Fig 5 Geometrv for capture by charged wire

Appendix: Capture Cross Section of a Charged Wire

We shall assume the following physical model with the mass of the particle much smaller than the mass of the wire so that the center of mass of the system can be assumed to be the center of the wire The wire is assumed to be moving with a velocity v perpendicular to its axis (see Fig 5) The Langrangian is

$$L = KE - PE = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - qV \quad \text{(A1)}$$

For the potential we use

$$V = -(Q/2\pi\epsilon)\ln r + C = V_0 - (Q/2\pi\epsilon)\ln(2r/d) \quad \text{(A2)}$$

where we have chosen the arbitrary constant so that at $r = d/2$, the potential equals the potential of the cable For greater accuracy we should actually put a screened potential in here, but this will only lower the cross section

As usual, the angular momentum is constant, and for particles coming in from large distances (the velocity of the wire larger than the velocity of the particles) it is

$$l = dL/d\dot{\theta} = mr^2\dot{\theta} = mvb \quad \text{(A3)}$$

From conservation of energy it follows that

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m\dot{r}^2 + \tfrac{1}{2}(l^2/mr^2) - qV_0 + (Qq/2\pi\epsilon)\ln(2r/d) \quad \text{(A4)}$$

We are interested in the case where the distance of minimum approach of the incoming particle is less than $r = d/2$, because then the particle will strike the wire and be captured Notice that for $r = d/2$, the logarithm is zero so that we have left only the terms

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv^2[b^2/(d/2)^2] - qV_0 \quad \text{(A5)}$$

Solving this for the parameter b,

$$b^2 = (d/2)^2[1 + (2qV_0/mv^2)] \quad \text{(A6)}$$

we can then use it to calculate the effective capture cross section of the moving wire

$$\sigma = 2Lb = Ld[1 + (2qV/mv^2)]^{1/2} \quad \text{(A7)}$$

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